LESSON 2-1 Using Inductive Reasoning to Make Conjectures

Find the next item in each pattern.

1. 100, 81, 64, 49, . . .

2. 6, 9, 12, . . .


4. west, south, east, . . .

Complete each conjecture.

5. The square of any negative number is ________________.

6. The number of diagonals that can be drawn from one vertex in a convex polygon that has $n$ vertices is ________________.

Show that each conjecture is false by finding a counterexample.

7. For any integer $n$, $n^3 > 0$.

8. Each angle in a right triangle has a different measure.

9. For many years in the United States, each bank printed its own currency. The variety of different bills led to widespread counterfeiting. By the time of the Civil War, a significant fraction of the currency in circulation was counterfeit. If one Civil War soldier had 48 bills, 16 of which were counterfeit, and another soldier had 39 bills, 13 of which were counterfeit, make a conjecture about what fraction of bills were counterfeit at the time of the Civil War.

One-third of the bills were counterfeit.

Make a conjecture about each pattern. Write the next two items.

10. 1, 2, 2, 4, 8, 32, . . .

11. ____________  ____________
**Lesson 2-1: Using Inductive Reasoning to Make Conjectures**

**Practice A**

Find the next item in each pattern.

1. 2, 4, 6, 8, . . .
3. fat, winter, spring, . . .

- 10
- W
- summer

4. When several examples form a pattern and you assume the pattern will continue, you are applying inductive reasoning.
5. A statement you believe to be true based on inductive reasoning is called a conjecture.

For Exercises 6–8, complete each conjecture by looking for a pattern in the examples.

- 6. The sum of two odd numbers is even.
- 7. The number of sides of a polygon that has \( n \) vertices is \( n \).
- 8. When a tree is cut horizontally, a series of rings is visible in the stump. Make a conjecture about the number of rings and the age of the tree based on the data in the table.

<table>
<thead>
<tr>
<th>Number of Rings</th>
<th>3</th>
<th>15</th>
<th>22</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age of Tree (years)</td>
<td>3</td>
<td>15</td>
<td>22</td>
<td>60</td>
</tr>
</tbody>
</table>

The number of rings in a tree is the same as the tree’s age.

9. Assume your conjecture in Exercise 8 is true. Find the number of rings in an 82-year-old oak tree.
10. A counterexample shows that a conjecture is false.


determine each conjecture is false by finding a counterexample.

- 11. For any number \( n \), \( 2n > n \).
- 12. Two rays having the same endpoint make an acute angle.

**Practice B**

Find the next item in each pattern.

1. 100, 81, 64, 49, . . .
2. 36
4. west, south, east, . . .

- Arkansas

Complete each conjecture.

- 5. The square of any negative number is positive.
- 6. The number of diagonals that can be drawn from one vertex in a convex polygon that has \( n \) vertices is \( \frac{1}{2}n(n - 3) \).

Show that each conjecture is false by finding a counterexample.

- 7. For any integer \( n \), \( n^2 > 0 \).
- 8. Each angle in a right triangle has a different measure.
- 9. For many years in the United States, each bank printed its own currency. The variety of different bills led to widespread counterfeiting. By the time of the Civil War, a significant fraction of the currency in circulation was counterfeit. If one Civil War soldier had 48 bills, 16 of which were counterfeit, and another soldier had 39 bills, 13 of which were counterfeit, make a conjecture about what fraction of bills were counterfeit at the time of the Civil War.

One-third of the bills were counterfeit.

Make a conjecture about each pattern. Write the next two items.

10. 1, 2, 4, 8, 16, . . .
11. Each item, starting with the third, is the product of the two preceding items: 256, 8192.

**Practice C**

Make a conjecture about each pattern. Write the next two items.

1. \(-1, -8, -27, -64, \ldots\)
2. \(1, 11, 21, 121, 11121, \ldots\) (Hint: Try reading the numbers aloud in different ways.)
4. \(_{3, 6, 10, \ldots, n}

First rotate the figure 180°. Then reflect the figure across a vertical line. Repeat.

Determine if each conjecture is true. If not, write or draw a counterexample.

- 5. Three points that determine a plane also determine a triangle.

- 6. An image reflected across the \( x \)-axis cannot appear identical to its preimage.

- 7. If \( a > b \) and \( b > c \), then \( a > c \).

- 8. If \( n \) is an integer (\( n > 0 \)), then \( \frac{1}{n} \leq \left( \frac{1}{n} \right)^2 \).

- 9. Hank finds that a convex polygon with \( n \) sides has \( n - 3 \) diagonals from any one vertex. He notices that the diagonals from one vertex divide every polygon into triangles, and he knows that the sum of the angle measures in any triangle is 180°. Hank wants to develop a rule about the sum of the angle measures in a convex polygon with \( n \) sides. Find the rule. (Hint: Sketching some polygons may help.)

**Reteach**

When you make a general rule or conclusion based on a pattern, you are using inductive reasoning. A conclusion based on a pattern is called a conjecture.

**Pattern**

<table>
<thead>
<tr>
<th>Term</th>
<th>Conjecture</th>
<th>Next Two Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-8, -3, 2, 7, \ldots)</td>
<td>Each term is 5 more than the previous term.</td>
<td>(7 + 5 = 12) (12 + 5 = 17)</td>
</tr>
</tbody>
</table>

The measure of each angle is half the measure of the previous angle.

Find the next item in each pattern.

1. \(\frac{1}{4}, \frac{1}{3}, \frac{1}{2}, 1, \ldots\)
2. 100, 81, 64, 49, . . .

- 36

Complete each conjecture.

- 5. If the side length of a square is doubled, the perimeter of the square is ________ doubled.
- 6. The number of nonoverlapping angles formed by \( n \) lines intersecting in a point is ________ 2n.

Use the figure to complete the conjecture in Exercise 7.

- 7. The perimeter of a figure that has \( n \) of these triangles is ________ \( n + 2 \).