LESSON 3-3
Proving Lines Parallel

Use the figure for Exercises 1–8. Tell whether lines \( m \) and \( n \) must be parallel from the given information. If they are, state your reasoning. (*Hint: The angle measures may change for each exercise, and the figure is for reference only.*)

1. \( \angle 7 \equiv \angle 3 \)

2. \( \angle 3 = (15x + 22)^\circ, \angle 1 = (19x - 10)^\circ, x = 8 \)

3. \( \angle 7 \equiv \angle 6 \)

4. \( \angle 2 = (5x + 3)^\circ, \angle 3 = (8x - 5)^\circ, x = 14 \)

5. \( \angle 8 = (6x - 1)^\circ, \angle 4 = (5x + 3)^\circ, x = 9 \)

6. \( \angle 5 \equiv \angle 7 \)

7. \( \angle 1 \equiv \angle 5 \)

8. \( \angle 6 = (x + 10)^\circ, \angle 2 = (x + 15)^\circ \)

9. Look at some of the printed letters in a textbook. The small horizontal and vertical segments attached to the ends of the letters are called *serifs*. Most of the letters in a textbook are in a serif typeface. The letters on this page do not have serifs, so these letters are in a sans-serif typeface. (*Sans* means “without” in French.) The figure shows a capital letter \( A \) with serifs. Use the given information to write a paragraph proof that the serif, segment \( HI \), is parallel to segment \( JK \).

**Given:** \( \angle 1 \) and \( \angle 3 \) are supplementary.

**Prove:** \( HI \parallel JK \)
Practice A

Proving Lines Parallel

1. If two coplanar lines are cut by a transversal so that a pair of corresponding angles are congruent, then the two lines are parallel.

Use the figure for Exercises 2 and 3. Given the information in each exercise, state the reason why lines a and b are parallel.

2. \(\angle 4 \cong \angle 5\)
   
   - **Reason:** Converse of the Corresponding Angles Postulate
   
   Conv. of Corr. \& Post.

3. \(m\angle 3 = 68^\circ\), \(m\angle 7 = 68^\circ\), \(\angle 3 \cong \angle 7\), Conv. of Corr. \& Post.

Fill in the blanks to complete these theorems about parallel lines.

4. If two coplanar lines are cut by a transversal so that a pair of alternate interior angles are congruent, then the two lines are parallel.

5. If two coplanar lines are cut by a transversal so that a pair of same-side interior angles are supplementary, then the two lines are parallel.

6. If two coplanar lines are cut by a transversal so that a pair of alternate exterior angles are congruent, then the two lines are parallel.

7. Shu believes that a theorem is missing from the lesson. His conjecture is that if two coplanar lines are cut by a transversal so that a pair of same-side exterior angles are supplementary, then the two lines are parallel. Complete the two-column proof with the statements and reasons provided.

- **Given:** \(\angle 1\) and \(\angle 3\) are supplementary.
- **Prove:** \(m\angle 1 = m\angle 3\)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
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<tbody>
<tr>
<td>1. (\angle 1) and (\angle 3) are supplementary.</td>
<td>a. Given</td>
</tr>
<tr>
<td>2. (\angle 2) and (\angle 3) are supplementary.</td>
<td>2. Linear Pair Thm.</td>
</tr>
<tr>
<td>3. (\angle 2 = \angle 3)</td>
<td>b. Given</td>
</tr>
<tr>
<td>4. (m\angle 1 = m\angle 3)</td>
<td>4. Conv. of &amp; &amp; Post.</td>
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</tbody>
</table>

Practice C

Proving Lines Parallel

1. a. \(m\angle 1 = 60^\circ\), \(m\angle 2 = 120^\circ\), \(m\angle 3 = 120^\circ\)
   
   b. \(m\angle 1 = m\angle 2 = 57^\circ\), \(m\angle 3 = 57^\circ\), \(m\angle 1 = 57^\circ\)

2. Use the figure and the given information to write a paragraph proof that the sum of the measures of the three angles in a triangle is 180°.

   - **Given:** \(\triangle ABC\)
   - **Prove:** \(m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ\)

   **Possible answer:** Construct \(\overline{FG}\) through point \(C\) and parallel to \(\overline{AB}\).

   **Possible answer:** If two coplanar lines are cut by a transversal so that a pair of corresponding angles are congruent, then the two lines are parallel.

   **The measures of the segments are equal.**

3. Construct another isosceles triangle with angles of 12° and 12°.

   - **Possible answer:** If a triangle is isosceles, then the sides opposite the congruent angles are congruent.